ON THE LIMITATIONS AND PROMISE OF QUANTUM THEORY FOR COMPREHENSION OF HUMAN KNOWLEDGE AND CONSCIOUSNESS

by Carl S. Helrich

Abstract. I present a partially historical discussion of the basis of the quantum theory in nonmathematical terms using human knowledge and consciousness as an underlying theme. I show that the philosophical position in both classical and quantum theory is the experimental and mathematical philosophy of Isaac Newton. Because almost all the systems we deal with are multicomponent, we must consider the limitations and openness imposed by thermodynamics on our claims in both classical and quantum treatments. Here the reality of measurement stands in the way of any simple picture but also provides the basis for considerations of free will. Particular care is taken with the concepts of quantum measurement, entanglement, and decoherence because of their importance in the discussion.

Keywords: classical and quantum theory; consciousness; experiment; human knowledge; information theory; measurement.

Theoretical physics represents what we can know scientifically about the structure of the universe. Any discussion of theoretical physics, then, brings us into contact with the sources and limits of human knowledge. If we press far enough in a discussion of this nature we must finally encounter human consciousness. Unless we accept a complete separation of the physiological brain from the mind, our theoretical physics must be capable of engaging free will and consciousness. I consider our ability to discuss consciousness to be a deep underlying interest in our discussion and use that interest to set the direction for portions of the discussion.

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Questions arising in any consideration of the origins of human consciousness are of the deepest nature, so we may expect our deepest understanding of physics and of physical reality to be necessary in our engagement with these questions. Perhaps the ideal situation would exist if we had in our possession a physical theory that was complete and exact and that we could demonstrate that our understanding of the meaning of completeness and exactness allowed us to undertake the engagement with confidence. But this is not the case and has never been in the history of science. We must make do with the tools we have. We should expect these tools to be altered by the engagement, as has always been the case in the development of scientific understanding.

The questions, of course, have already been engaged in at least the last century. And the problem of free will was acknowledged by Isaac Newton, who recognized that this was incompatible with a mechanical picture of the universe (Park 1988). So we cannot pretend that we are at the beginning of a research program.

In this essay I briefly summarize what I think are important issues for the engagement. I first look carefully at Newtonian thought and what is in fact meant by experimental and mathematical philosophy as Newton expressed that. I then show the continuity of that philosophy with the modern ideas of the quantum theory. I specifically do not easily dismiss the ideas on which classical physics is built. I speak to the fundamental issues of the quantum theory, quantum measurement, entanglement, and decoherence. My treatment is not exhaustive. I try to briefly provide the issues in pure form. I discuss some of the basic ideas of the quantum theory and related aspects of theoretical physics. I do this without recourse to mathematical equations. Some mathematical concepts are necessary for clarification, but I make those as transparent as possible verbally.

At no point do I move easily from what we understand about the quantum or the classical theory into speculation. Speculation and human judgment are an integral part of all scientific and theological investigations, but it is very important that we be aware of the points at which we are indulging in speculation and how secure, if at all, that speculation may be. Scientific knowledge is not the only knowledge. And scientists do use their own desire for beauty and their own metaphysical positions in that judgment. Our final arbiters, however, are the laboratory and the demand for consistency. We violate these at our own risk.

If we are to attempt an encounter with anything resembling reality we must ask questions about the depth of our engagement with nature. We presently possess a set of laws represented by mathematical equations, which describe experiments. In that sense they are descriptive equations. If these equations were final or ultimate truth, or even if they were a representation of that truth, we could claim that these equations were prescriptive. A question of the existence of ultimate truth is, in a scientific sense, a question about the existence of such a set of prescriptive equations. Our belief or lack of belief in the existence of such a set of prescriptive equations is a metaphysical stance.

My own metaphysical position, as a theoretical physicist, is a belief in the existence of a prescriptive set of equations. I do not believe, however, that we shall ever be able to write those equations on a piece of paper. The mathematics on which they may be based is probably also beyond our abilities at this time, for reasons I describe below.

EXPERIMENTAL AND MATHEMATICAL PHILOSOPHY

The question of how we go about defining reality is primary to any scientist, philosopher, or theologian. This has never been an easy question to answer. Enlightenment science is characterized by the picture of the universe given to us by Isaac Newton. This picture includes a concept of physical reality based on the motion of material bodies that seems easily understood in terms of our commonsense ideas of measurement, motion, space, and time. There were critical problems, which Newton recognized very well and were the basis of his intense interest in alchemy (Cropper 1995). Anyone who thought that atoms and the void formed the basis of the universe confronted the question of life, human nature, and the concept of God. Newton also was concerned because he thought that the planetary system was mechanically unstable. He believed that God needed to apply something of a guiding hand to this to hold the balance. Gottfried Leibniz derided Newton for this (Park 1988), and later studies in mechanics by Pierre Simon de Laplace showed that even within the Newtonian mechanics the planetary system was stable.

The picture with which we are most familiar as a generality of Enlightenment thought is that of Laplace, who asked us to

> Assume an intelligence which at a given moment knows all the forces that animate nature as well as the situations of all the bodies that compose it, and further that it is vast enough to perform a calculation based on these data. It would then include in the same formulation the numbers of the largest bodies in the universe and those of the smallest atom. For it nothing would be uncertain, and the future, like the past, would be present before its eyes. (Laplace 1843–47, vol. 7 p. vi, quoted in Park 1988, 393)

At the time these words were written this was the conclusion that could be reached based on the analytical mechanics of Joseph Louis Lagrange and Leonhard Euler, based on Newton's laws.

The paradigm consisting of Newtonian mechanics, the electromagnetism of James Clerk Maxwell, and the thermodynamics of Rudolf Clausius and William Thomson constitute what physicists term classical physics. Philosophers refer to this as the modern paradigm.

Underlying this classical, or modern, paradigm is a fundamental position held by Newton, and his mentor Isaac Barrow, as the principle of a mathematical and experimental philosophy. The roots of this are found with Nicolaus Copernicus, Galileo Galilei, and Johann Kepler. Barrow rejected the scientific method of Francis Bacon (*Novum Organum* [1620] 2000), which was to lead mechanistically to truth, based on a tabulation of facts and the generation of hypotheses using induction. It was the generation of hypotheses that was specifically a problem. Barrow and then Newton wanted an inductive method that led from experimental observations to laws based on mathematical principles.

In the Newtonian picture a theory can be described verbally. And Newton's four laws, which constitute the Newtonian theory of mechanics, were presented in the *Principia* first in verbal form. However, in its fundamental form the theory is contained in the set of equations obtained inductively from experiments. Validity of these mathematical laws cannot be guaranteed. And to some the result appeared to be a hollow shell, because this mathematical structure did not contain the sort of detailed picture proposed by, for example, René Descartes (Park 1988, 185).

Maxwell's electrodynamics is exemplary of this approach. In spite of the mechanical pictures Maxwell had to describe the action of a supporting luminiferous aether, Maxwell's equations and the waves they predicted existed independently of that picture and even, as Albert Einstein showed (1905a), independently of the luminiferous aether. The theory told us what to look for—the waves—and gave us the interpretation of the experimental results. An ingredient that Bacon missed in the attempt to develop a methodology was the importance of judgment in the scientific enterprise.

CLASSICAL ENCOUNTER WITH CONSCIOUSNESS

At the end of the nineteenth century William James pointed out that conscious thought could not be produced from the molecular and cellular structure of the brain (Stapp 1993). His argument was based on the deterministic reductionism of the classical theory. Because there is free will there must be something beyond the determinism of the mechanical description.

Henry Stapp contends that the problem is that classical physics is reductionist in the sense that the classical physicist accepts as the only real things in nature material particles or fields (electromagnetic, gravitational). Stapp points out that a conscious thought is itself a unity that cannot be an aggregation of simpler things and that nothing in classical physics can produce anything that is more than an aggregate of its parts. On this he builds his bridge to the quantum theory, which he claims naturally fulfills this requirement.

If this were absolutely true, classical physics would be incapable of considering consciousness or free will. But there is a very important part of classical physics that is neglected in this argument: thermodynamics. To go from the classical mechanics of particles to the human brain requires that we engage the science that deals with matter and its interaction with energy, which is thermodynamics.

The two great principles of thermodynamics were established during the period when some of the principal contributors, such as Rudolf Clausius, were also working on the kinetic theory of gases. Clausius was, however, very careful to not mix kinetic theory with thermodynamics, because thermodynamics had to stand on its own experimental foundations (Brush 1965, 24). This was particularly important in the case of the second law, of which Clausius was a primary author.

Demonstrating a unity of kinetic theory and thermodynamics was a necessary step in establishing the reality of atoms, and the problem of doing so was crucial. The second law of thermodynamics is the only law in classical physics that specifies a directionality for time. The entropy increases during irreversible processes. Newton's second law of motion is time-reversible, and an irreversible result cannot be obtained by any reductionist combination of deterministically connected molecules. Quantum theory also does not solve this problem, because classical theory is rigorously obtainable from quantum theory (Omnès 1999, 81).

In the last Herculean scientific act of his life Josiah Willard Gibbs produced the manuscript of his treatise on statistical mechanics, which contained a formulation of the entropy in terms of an average over an ensemble of systems. Gibbs's formulation of entropy is correct thermodynamically. The Gibbs and the James families were well acquainted (Rukeyser 1942), and James sought to make psychology more of a science. But there is no reason to suppose that James was familiar with the intricacies of Gibbs' arguments. A century after its (1902) publication, Gibbs's statistical mechanics is still often considered inaccessible But in Gibbs's formulation of the second law, which encompasses the previous formulation of Ludwig Boltzmann, we find a possible key to the problem confronting James, and confronting us, regarding an accounting for the mind and free will in terms of physics. The key is measurement.

The final argument that established the existence of atoms was Einstein's treatment of Brownian motion, which was based on the Boltzmann formulation of the second law (Einstein 1905b).

Gibbs's concept of the ensemble accepted the impossibility of making measurements on individual interacting molecules in large systems. The number of molecules in any laboratory system is of the order of 10^{23}. And we know the values of only a handful of physical parameters for the system. We cannot then claim that even classical physics provides a passage from a detailed picture based on the mechanics of objectively real material molecules to thermodynamics. The reality of measurement stands in the way.

Classical physics does implicitly claim that material particles have an objective reality. As long as we can see these particles with our eyes, or even under our microscopes, we are prepared to claim that they obviously occupy space and have momentum, whether we take the trouble to measure those or not. Classical physics also does not make the details of the measurement process integral to the theory. The most elegant formulation of classical mechanics, that of William Hamilton and Carl Jacobi, claims complete knowledge of all positions and all momenta of a system at a particular instant of time with no consideration of how the measurements of those quantities are to be obtained. But to make claims about the positions and momenta of the molecules of a system is to pass beyond the limits of the experimental and mathematical philosophy of Newton. We must then consider that these claims to knowledge of the details of the system actually constitute a statement in addition to the precepts of experimental and mathematical philosophy.

Is there any real possibility of free will and of consciousness if we use only classical physics? One of the greatest physicists of the twentieth century believed there was. Max Planck wrote on the question of free will in the first part of the twentieth century. Although Planck was quite aware of the emerging quantum theory and was the first to propose a quantum of action, his thoughts on the problem of free will were classical. Planck considered himself a thermodynamicist and a disciple of Clausius. He pointed to the issue of measurement and self-reference as the seat of the problem of free will (Heilbron 1986).

I personally believe that this is still the primary issue in any attempt to obtain, in scientific terms, an understanding of either consciousness or free will. This has become more elusive and more conceptually difficult than what Planck considered. We also know far more about the biophysics of the brain than we did almost a hundred years ago. But it seems to me that the basic problem has not changed.

MODERN PHYSICS

The twentieth century brought us deeper understanding of what measurement means, and a sharpening of the basic Newtonian experimental and mathematical philosophy, but no fundamental change in the basic tenets of this philosophy. The twentieth-century scientist Paul A. M. Dirac wrote in his classic treatise on quantum mechanics that “Only questions about the results of experiments have a real significance and it is only such questions that theoretical physics has to consider” (Dirac 1958, 5).

Dirac’s specific point is that theoretical physics does not propose to explain those experiments in terms of any picture of a reality lying beneath the experiments. This again is Newton in very stark and terse terms.

This refusal to consider an underlying picture is not equivalent to the position of the logical positivists, who claimed that no structured mathematical picture existed at all and that we could only make observations. They denied the possibility of a theoretical physics (Mach 1942).

In an address titled “Mathematical Problems” delivered to the Second International Congress of Mathematicians in Paris in 1900, David Hilbert ([1900] 2000) defined twenty-three unsolved problems confronting mathematicians at the beginning of the new century. Number six among these was to axiomatize all of physics. It is fairly clear that we have not accomplished what Hilbert expected, principally because the physics that emerged in the twentieth century was not the physics Hilbert believed existed. But we have based our understanding of the physical universe very firmly in our mathematics. The words of twentieth-century physicist Eugene Wigner speak to this: “The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve” (Wigner 1960, 14).

Dirac added to this the belief that the equations of physics must be beautiful. In his own words, “It often happens that the requirements of simplicity and beauty are the same, but where they clash the latter must take precedence” (Dirac, quoted in Cropper 2001, 373).

Mathematics, however, presents us with a mystery that we have no way of resolving and with a limitation to our understanding that we must accept. The mystery is whether mathematics is a product of the human mind, freely developed, or is something existing beyond us that we are discovering. In his words here Wigner provides no answer to the dilemma, but he acknowledges the depth of the mystery. Mathematics is the language of the universe, and we have no way of explaining why.

The limitation we know we face is that spoken to by Kurt Gödel in the first of his incompleteness theorems published in 1931. In this theorem Gödel proved that any axiomatic system containing an algebra is internally inconsistent. That is, there are propositions that may be true, but they cannot be established from the axioms.

We cannot then claim an understanding of the universe based on a complete, axiomatic mathematics. We have not yet, however, come up against any evident limitations based on incompleteness. As far as our present investigations have taken us we can still claim the miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics.

The agreement of the predictions of quantum theory with our experimental measurements is remarkable. If quantum theory provides a mathematical structure that is at least close to that of the universe, it seems to follow that the basis for an understanding of consciousness may be buried in that structure. The quantum theory has specifically given us an example of a physics that relies on a complete adherence to experiment and a mathematical formulation.

If we want to consider the mind and the physiological brain we must treat systems composed of collections of subsystems. This has aspects that I believe bear directly on the present discussion. Specifically I want to consider quantum entanglement, decoherence, and the ideas of John von Neumann and Wigner.

QUANTUM MEASUREMENT

For clarity we must consider quantum measurement. The act of measurement, whether quantum or classical, utilizes an apparatus that is set up for the measurement we wish to make. For a classical system the range of values we may receive for the measurement is continuous. A quantum measurement returns a single number from a set of two or more possible numbers. The result of a measurement must, therefore, be a real number, because a complex number requires two components for its definition. To each number obtained in the measurement process there corresponds a particular value of some property of the system, the definition of which is specified by the quantum theory. The state of a system is known when the values of the knowable properties are determined. So a measurement specifies a state or a subset of states of the system.

If the same measurement is repeated immediately on a quantum system the same result is returned. The measurement act does not disturb the state if the system is in a particular quantum state to be determined by the measurement.

Before the measurement we do not know the quantum state of the system at all. We know only that the measurement we are about to perform will return a single number that is specific to the measurement. The only way in which this situation can be realized is if the state of the quantum system before the measurement is represented as a linear sum of the possible states. For example, let us suppose that we want to measure a quantity, which we shall designate as q. We shall assume that the outcome of the measurement can be either q_0 or q_1 corresponding to the states of the system, which we represent symbolically as $|0>$ and $|1>$. These symbols designate what are called state vectors. Then the state of the system before measurement is $\alpha|1> + \beta|0>$ in which α and β are generally complex numbers with real and imaginary components. Max Born showed that the squares of the moduli of α and β, which are real, are proportional to the probabilities[1] that the measurement will yield q_0 or q_1 (Born 1926). The quantum theory tells us how to obtain q_0 and q_1 from $|0>$ and $|1>$ and how to find the time evolution of the quantum state. The quantum theory itself is not based on probabilities.

Probabilities enter regarding the possible outcomes of a measurement that we plan to make. If we change the nature of the measurement apparatus, that is, if we rotate it, our knowledge of the system before the measurement changes accordingly, because the possible outcomes are different. The measurement will now return any one of a new set of numbers corresponding to the new orientation of the apparatus.

Suppose, for example, that we have two apparatuses designed to measure the same quantum property, which are aligned along the path of a quantum system but rotated with respect to one another. Each apparatus will return numbers q_0 or q_1, which correspond to state vectors $|0>$ and $|1>$ for the first apparatus or q'_0 or q'_1 corresponding to the state vectors $|0'>$ and $|1'>$ for the second. The numerical values of q_0 and q_1 may be the same as or q'_0 and q', but the state vectors are distinct. Let us assume that the first apparatus returns the number q_1. Then the system leaving the first apparatus is in a state specified by the state vector $|1>$ with respect to that apparatus. However, the system will leave the second apparatus in a state that bears no definite relationship to this state. Particularly we cannot calculate the result of the second measurement from the first. We can only specify probabilities for outcomes of the second measurement.

This is straightforward quantum theory. But it denies objective reality in the classical metaphysical sense. I use here the term metaphysical to indicate that even the classical claim of objective reality cannot be established experimentally in the case of small particles. Reality is, however, more akin to what we presently have as quantum reality than to classical objective reality.

OBJECTIVE REALITY

The question of an objective reality was central to one of the great discussions of the twentieth century between Niels Bohr and Einstein. Bohr's position was basically that of the quantum reality described in our discussion here, although he cast it in terms of the principle of complementarity. Einstein's position was that an objective reality must exist and that the quantity measured is actually possessed by the quantum system before the measurement.

Einstein's final challenge to the quantum theory was in his paper written with Boris Podolsky and Nathan Rosen (Einstein et al. 1935). In this four-page paper, known as the EPR paper, the claim was made that quantum theory was incomplete because of its inability to account for an objective reality. The paper cast the question in terms of two quantum systems emerging from the same physical point. Specifically the proposal was that the quantum systems *I* and *II* interact from a time $t = 0$ to a time $t = T$, after which they separate. This made the system what we now term an entangled quantum system. Entangled quantum systems are the objects of considerable research at this time because of their relation to quantum computing and cryptography (Braunstein and van Loock 2005).

Bohr could provide no adequate refuting argument to the EPR paper. In 1964 John Stewart Bell was able to specify an experimental measurement that could decide between the quantum theory of measurement described above and a quantum theory developed by David Bohm, in which the measured quantities were determined by a hidden variable possessed by the quantum system (Bell [1964] 1987; Bohm 1952; Helrich 2000). The experiment was designed to measure a difference in correlation functions. The results of this measurement would form the basis upon which a decision could be made in favor of either the quantum theory described here or one based on hidden variables and an objective reality. This was a falsifying experiment, because the measurement produced numbers that could be compared and from which comparison a clear conclusion could be reached regarding the validity of one or the other approach. No philosophical discussion was necessary or desirable. In 1982 the experiment was conducted by Alain Aspect (1982a, b). The result was in favor of quantum measurement.

The theoretical physicist N. David Mermin (1985), with some humor, classifies physicists according to their position on and desire to understand the implications of the quantum theory and the measurements of Aspect. But Mermin and Chris Isham note that the issue is not at an end. In the epilogue to his book *Lectures on the Quantum Theory*, Isham writes that “it would be nice to finish with a chapter entitled ‘The solution to the conceptual problems’, but unfortunately that is not possible; at least, not in any really comprehensive sense” (Isham [1995] 2001, 219). Roland Omnès devotes much of his book *Understanding Quantum Mechanics* (1999) to the question of interpretation of the theory and is finally explicit about the fact that we have only a present position on interpretation and not a final interpretation.

This stirs our imagination. And our ability to imagine is an integral part of physical theory. I believe, as others have before me, such as Planck, that physics itself provides an openness that will allow investigation of the basis of conscious thought and free will. In the language of some, this may provide insight into the interaction we have with God and perhaps some understanding of the possibility of revelation. We must, however, be very careful that we do not move into a realm in which we violate what is already known about the physics. We also must take seriously Paul Tillich’s claim (1951) that God is infinite mystery and not a problem to be solved.

KNOWLEDGE

Our scientific knowledge of a system, whether quantum or classical, is based on measurement. Only in quantum physics, however, have we considered the limitations of measurements of individual quantum systems and the consequences of those limitations.

In classical physics this analysis is replaced by the assumption that we know the state of the system without any detailed analysis of how that can be possible. The results of a classical analysis provide final positions and momenta defined as exactly as the initial conditions. A detailed consideration of the actual limitations associated with measurements based on possible arrangements of apparatus have been considered in modern chaos theory. We now know something of the wide variations in final conditions that can result in classical systems from extremely small variations in initial conditions. In the sense of an honest evaluation of what we can and cannot know about the classical system based on possible measurements, classical mechanics also possesses an openness.

In neither quantum nor (chaotic) classical descriptions do we require explicitly the human mind. What is required is an honest analysis of the limitations of human knowledge based on the possibilities of physical measurement.

The quantum states identified with the vectors $|0>$ and $|1>$ may provide more knowledge about the system than that gained by the measurement undertaken to decide between them. In the quantum theory there is a mathematical operator associated with each physical property of a system. The vectors $|0>$ and $|1>$ are mathematical quantities associated with possible values q_0 and q_1 of a physical property. If we designate the operator associated with the variable q as Q, the operation of Q on $|0>$ produces q_0 and the operation of Q on $|1>$ produces q_1. That is, $Q|0> = q_0|0>$ and $Q|1> = q_1|1>$. In mathematical terminology the vectors $|0>$ and $|1>$ are eigenvectors of the operator Q, and the numbers q_0 and q_1 are eigenvalues of the operator corresponding to those eigenvectors. The vectors $|0>$ and $|1>$ may also be eigenvectors of other operators R, S, . . . which specify values or ranges of values of the corresponding properties r, s, . . . The measurement of Q is then sufficient to specify the values of, or ranges of values of, the physical properties corresponding to R, S, . . . as well. The state vector then contains knowledge of the physical properties of the quantum system corresponding to Q, R, S, . . .

Consider that we perform a measurement of the property corresponding to either Q or R. After the measurement the state vector of the system is an eigenvector of the operator chosen. Let us assume we choose to measure the property corresponding to the operator R. We immediately follow this by a measurement of the property corresponding to the other operator Q. The measurement "R then Q" we indicate by the operation QR. Because the state vector of the system leaving the measuring apparatus for R is a pure eigenvector of the pair of operators Q and R, the measurement of the property corresponding to Q will not alter this state vector. Identical considerations follow for the measurement "Q then R," which is designated as RQ. Either sequence of measurements produces the same state of our knowledge about the system. Therefore, $QR = RQ$ if the operators have the same eigenvectors. The operators then commute. The result is mathematical but has a clear physical meaning. The state vector contains all that can be known about the system. The state vector itself has, however, no physical meaning.

The state vector is an abstract quantity in the mathematical sense. This sense is essentially no different from the sense of abstract used in ordinary discourse. We can deal with an abstract mathematical quantity only if we represent it in some more familiar terms. If we represent the abstract state vector in terms of space and time we have the Schrödinger wave function Ψ.

The principal equation of nonrelativistic quantum theory is the Schrödinger equation. Written in abstract form it is extremely simple. Central in the equation is the Hamiltonian operator, which contains the interaction among all the quantum systems pertinent to the situation being considered. If we represent the Schrödinger equation in space and time it takes on the form of a partial differential equation whose solution yields the wave function Ψ. This is the form in which Schrödinger first obtained and published it (Schrödinger 1926).

The wave function Ψ is generally a complex valued function, which has both real and imaginary parts. Schrödinger initially thought that Ψ was an actual physical quantity and that what we knew as particles were really coalesced waves. The principal difficulties with Schrödinger's interpretation were the complex valued property of the wave function and the fact that the wave function is spread out over a region of space. Measurements of electrons in diffraction experiments, for example, provided single spots on photographic films, which resulted from impacts of single electrons with atoms making up the film. The results were not smeared out over regions of the film. Max Born, who provided the interpretation of the wave function, remarked that he could go down the hall to look at James Franck's experiments in which particles left tracks in cloud chambers (Messiah [1958] 1999). Born pointed out that, although the wave function itself has no physical meaning, the square of the modulus of the wave function is the probability density function for the spatial location of the quantum system.

For applications we are interested in obtaining a description of the possible quantum transitions in a particular system to provide an understanding of experimental results or to gain insight into possibilities. We may choose any one of a number of possible sets of commuting variables on which to base our analysis. That choice determines which other variables we may also know based on the commutation relations among the operators for those variables. These commutation relations are dependent upon the interactions among parts of the quantum system. The choice is always made on the basis of mathematical simplicity and the objectives of the problem at hand. Once a choice of one set of commuting variables is made for the analysis, another set of variables is left as undetermined. This is the basis of Bohr's principle of complementarity. If we choose to make a particular measurement we give up the possibility of making a complementary measurement.

ANALYSIS AND APPLICATION

The analysis of a quantum system provides all we can know about the system. The analysis may, for example, be integral to a particular laboratory experiment. Theory, as Einstein pointed out, tells us what experiments mean. Our analysis will be based on a set of mathematical operators constructed to account for the interactions among the components of the quantum system. These are usually particles and fields. The states available to the quantum system are determined by the operators and depend on the particles and the fields present.

Any analysis of multicomponent systems must also be based on the methods of Gibbs's statistical mechanics applied to quantum systems. This will bring in questions of entanglement and decoherence.

We have the freedom to choose the eigenvectors of any commuting set of operators as the basis for our analysis. We normally choose the simplest possible basis for our analysis. But the form of the operators results from the mutual interactions of the components of the system, so our choice of a basis depends strongly on the situation at hand. The results of our analysis may be presented as tables of numbers representing possible values of physical properties corresponding to our choice of operators.

There is a sticky point here. We have used a set of eigenvectors as a basis. We can do that only with expectation that our results will be at least close to reality if the set of eigenvectors is mathematically complete. Is it? This is something we cannot prove mathematically. It is a fundamental postulate of the quantum theory. We believe that it is correct, and seldom do we think about this in our work. It is a point at which Gödel's incompleteness is evident. Here we make a postulate we believe to be true without proof based on the preceding structure.

We may make statements about the probabilities that a particular quantum system is in a particular state by considering that system to be part of an ensemble of systems and applying the ideas of Gibbs's statistical mechanics. But we can say nothing definitive about a single system.

In this we may be reminded of Pythagorean numerology. It seems as though numbers have taken precedence over any description of the physical system. As in all of physics, we must be careful, however. Our table of numbers has resulted from an arbitrary choice of a basis. Had we chosen a different basis we would have obtained a different set of numbers. The approach we have used also relieves us of the necessity of obtaining any representation whatsoever of the state vector, which is a mathematical entity regardless of a lack of physical meaning.

In this discussion we have introduced what is termed a Heisenberg rather than a Schrödinger picture. They are mathematically equivalent. In the Schrödinger picture the basis is the time-dependent state vector, which represents all we can know about the system as it evolves in time. In the

Heisenberg picture the basis is a set of time-independent eigenvectors, which represent all we can know about the states available to the system in the course of its evolution in time. If we are interested in time development of a system our table of numbers will be dependent on the picture we choose as well as on the set of eigenvectors we choose. These choices are arbitrary.

In some applications we want to obtain an understanding of spatial positions of quantum systems. Before a measurement is made all we can know or speak about is the probability of locating the system in some region of space. The square of the modulus of the wave function $|\Psi|^2$ is the probability density function for the location of the system. The probability that the system is located in a particular small volume of space is the product of this probability density function with the small volume.

The probability density function is not the quantum system, which may, for example, be an electron. Any measurement of the location of the electron will reveal a particle. However, any mental picture we may try to construct of an electron orbiting the nucleus is not reality. The square of the modulus of the wave function represents only the knowledge we can have about the electron's position before a measurement is made. The wave function Ψ has no physical reality, and squaring its modulus produces no more physical reality.

It is instructive to exhibit plots of a particular value of the probability density function for electron states in the hydrogen atom, and most textbooks on quantum mechanics or physical chemistry do so. Unfortunately students too often interpret these bulblike plots as pictures of the electron and begin to think of the electron as being smeared out in space as the plot seems to indicate.

The quantum theory tells us that a quantum system in a state for which the state vector is an eigenvector of the Hamiltonian will remain in that state provided the Hamiltonian does not depend on the time. Time-dependent Hamiltonians, such as those describing interactions with time varying fields (electromagnetic waves), can result in transitions among states for which the state vectors are eigenvectors of the Hamiltonian. This is a direct consequence of the Schrödinger equation, but it also is seen in the Heisenberg indeterminacy principle in terms of energy and time rather than the usual position and momentum. If the electron is in a specific quantum state for which the energy is exactly known, the indeterminacy in the time is infinite.

This may be observed in laboratory measurements of atomic or molecular spectra. The light observed is the result of transitions between energy states. The frequency of the light emitted is related to the energy of the transition by the Planck-Einstein formula. Each transition contributes a quantum to the electromagnetic radiation, which is the measured light. This quantum has been termed a photon. Because it is emitted at all, regardless of what may be considered to be the cause of the emission, the lifetime of a state has been finite. The energy is then, by Heisenberg's principle, indeterminate, and a broadening of the spectral line results.

Other effects, such as intermolecular collisions and the motion of nuclei, will broaden the spectral line. These provide probes for such things as star temperatures. But these applications are not our primary concern here.

If we want to actually measure the orbit of the electron in a molecule we are faced with a dilemma. To measure the orbit we must obtain measurements of the position of the electron at two different times. To measure the location of the electron we must use a photon of light from a source and measure how the photon is reflected after colliding with the electron. The wavelength of the light to be used must be chosen to be short enough that the location of the electron within the orbit can be specified from the measurement. Unfortunately the resulting energy of the photon is sufficient to eject the electron from the molecule. This led Heisenberg to claim that the electron orbit has no place in the quantum theory, because it is not a measurable property (Heisenberg 1930).

DECOHERENCE

The precise definition of entanglement can be given in terms of state vectors for composite systems (Isham [1995] 2001). But for our purposes it suffices to recognize simply that quantum theory denies us the ability to consider the constituents of a composite system independently of one another. This dependence has roots in a quantity called *spin*, which has no classical analog, and the requirements placed on systems of identical particles by Wolfgang Pauli's exclusion principle. Entanglement is then an unavoidable consequence encountered in multicomponent systems. All physicists understand the implications of this, but no physicist can explain in any more basic terms why this principle exists.

The quantum interactive terms of entanglement become rapidly dissipated upon interaction with macroscopic systems through decoherence (Stoeger in Russell et. al. 2001). We can then not expect obvious quantum effects to be measured in biological systems. This does not diminish their potential importance in those systems. Nor does this imply anything fundamental about the importance of what quantum theory has taught us as we try to consider the functioning of the human brain. It does mean that in our search for quantum effects in the brain particularly we cannot easily dismiss certain aspects of physical theory without careful consideration.

HUMAN CONSCIOUSNESS

The role of human consciousness in the quantum theory has been of interest at various times. Von Neumann ([1932] 1955) introduced a theory of quantum measurement that involves the human brain in the last step. He assumed that the only physics was quantum physics and that, therefore, the interaction with a measuring apparatus was itself also quantum. The idea went together well if one considered only an ideal situation, but difficulties arose for realistic conditions and resulted in the propagation of quantum interferences (entanglements) to a macroscopic level. The resolution was to terminate the measuring process with an observer having consciousness. The individual's consciousness was claimed to be a unity, which was not subject to the multiplicities of the quantum theory (Omnès 1999).

The problem with this solution is that it uses unresolved aspects of human consciousness to resolve difficulties in physical theory. Although it is a rapidly progressing area of research (in which I am personally involved), our present understanding of the details of the biophysics of neural transmission is still incomplete. And, as with any research program, we do not know how far we are from complete understanding or even if that can ever be attained.

I have pointed out that the human mind is an unnecessary part of the measurement process. The state of Schrödinger's cat is not a linear superposition of alive and dead states to be resolved when we open the box. The quantum measurement was made by the Geiger counter, and the result was either nothing or an electrical pulse. It is, however, possible and even probable that our understanding of the quantum theory is an important ingredient in our attempts to understand the human brain and consciousness. The quantum theory represents our deepest picture of the universe that can be tested experimentally at this time. Its role in the most difficult of scientific problems, the study of human consciousness, may be logically expected. We may then ask questions in general terms about that possible role.

There are at least two aspects of the problem. The first is the physical—the physiological behavior of the brain. The second is the consciousness that results from the physical state of the brain. This consciousness is intimately linked to the dynamic electrical or electromagnetic state of the network made up by the neurons of the brain (Hopfield 1999; 1982; Levitan and Kaczmarek 1997; Cooper 1995). This state must be considered to be macroscopic and is supported by the physiological network of neurons and the connections among them. That is, the two parts of the problem are linked. The exact relationship between the electromagnetic state in the network of neurons and a conscious state is unknown.

We understand much of the basic physiology of neurons. We know that the electrical pulse that passes down the neuron results from the opening and subsequent closing of ion channels providing for the passage of sodium into and potassium out of the neuron as the electrical pulse passes down the axon. This pulse finally results in the release of calcium at the synapse, which induces a release of neurotransmitter to the receptors in the dendrite of the next neuron. Many of the details of this last step are not understood at this time. We have identified the principal proteins involved in the docking of the vesicles containing the neurotransmitter, but we do not know which protein, if any, is the target of the calcium or the process by which the pore is formed in the synapse for the release of the neurotransmitter. Any claim regarding a detailed picture of the functioning of even just the neurons in terms of physical theory is premature.

We may ask questions at a higher level regarding the function of the human brain in producing consciousness. Here we must engage the neural network, thermodynamics, and entropy production because of the irreversible nature of the phenomena. We also may choose to consider information theory.

Time is of primary importance in any attempt to relate the functioning of the brain to consciousness. The physical processes involved in neural transmission are highly nonequilibrium processes. In the resting state an electrical potential is maintained across the membrane of the neuron, which, because of the small membrane thickness, results in a high electric field within the membrane.[2] Upon excitation the product of this and the membrane current results in a relatively high entropy production rate in each individual neuron. Active brain processes are then irreversible in the extreme. I shall not speculate on whether or not our psychological concept of a time directionality is at all related to this aspect of the biophysics of the brain. The extreme irreversibility of these brain processes does, however, have consequences for our considerations of the role played by information in brain function.

I discussed the relationship between thermodynamics and information theory in a previous *Zygon* publication (Helrich 1999). My principal point in that discussion was that our present information theory can treat Markov processes but is inadequate for serious discussions of interconnected systems. Markov processes are often termed random walk processes and are characterized by the requirement that the result of each step is independent of all preceding steps. It seems clear to me that the physical processes taking place in the brain cannot be reasonably modeled by a Markov process and that any arguments based on our present mathematical formulation of information must be treated with healthy skepticism. These difficulties were already recognized by Edwin T. Jaynes (1957) in his second foundational paper on the application of information theory to statistical mechanics.

In its application to physical systems, the present form of information theory invites questions regarding the relationship between information and energy. Claude Shannon's formulation of information theory produced a mathematical expression for uncertainty that is mathematically identical to the formulation of the statistical entropy Boltzmann had obtained for dilute gases near equilibrium (Shannon and Weaver [1948] 1949; Boltzmann 1872).

Boltzmann was attempting to solve the problem of irreversibility in physical systems, which is the content of the second law of thermodynamics. The result, interpreted statistically rather than as an exact formulation of the problem, produced a quantity that was statistically monotonic in time—that is, irreversible. Central to Boltzmann's theory is an approximate formulation of the rate of collision between gas molecules, which ignores correlations among the molecules. Kinetic theorists are well aware of the problematic status of Boltzmann's results but use them because no better formulation is available, even though our understanding of irreversibility has increased considerably beyond that of Boltzmann.

As already noted, at equilibrium Boltzmann's formulation is contained within the more complete formulation of Gibbs. Gibbs's treatment requires no detailed description of molecular dynamics as long as we ask no detailed questions. Generally the equilibrium formulation of Gibbs applies to systems for which there is no entropy production and, therefore, no energy transport in any form. This is the final physical basis for the claim that information is separate from energy.

Information changes the uncertainty we have about a system. That is, the information contained in a process can be formulated from a difference in uncertainties, or statistical entropies, before and after the process, provided the system is in a state of thermodynamic equilibrium before and after the process. If we have no interest in the details of the process taking the system from one equilibrium state to the next, we can formulate the net information contained in the process. If we are considering the physiological brain and its relation to consciousness, however, our primary interest is in the irreversible process itself. But the brain is never in a state of complete equilibrium as long as the person is living. We must then obtain a formulation of information, or uncertainty, for irreversible processes if we are to understand consciousness in terms of information transfer or if we are to claim any understanding of the relationship between mind and matter in terms of information theory.

The results of such investigations may bring us insight into consciousness and into the basis of human perception of time, because time appears to be so important in the physical basis of consciousness. If quantum theory is also important in these investigations, which I suspect is the case, we may also obtain insight into what the quantum theory actually means. We cannot, however, assume what the future may bring, as Pauli pointed out on numerous occasions.

DISCUSSION

I have presented here an overview of some aspects of theoretical physics that contribute to our understanding of the quantum theory and consciousness. I am primarily a physicist and believe that my greatest contributions to the dialogue between religion and science can be realized if I remain in the position of a scientist. I can only attempt to clarify certain details of theoretical physics in the hope that an understanding of these details will be helpful to my colleagues in theology and philosophy. I do not consider myself competent to engage in the development of theological concepts.

I have shown that the line of demarcation between classical and modern physics is not as clear as some may believe. The philosophical position of the modern physicist is distinctly Newtonian, if we accept the terse statement of Dirac as the basis of our theoretical physics. We no longer accept certain added statements such as Newton's belief in an absolute space and time, however.

I also have revealed something of the role of thermodynamics in any considerations of the treatment of multicomponent systems. My point is that we must be very careful regarding our claims when they are based on the behavior of the individual components of these systems. The reality of measurement stands in the way of our speculation.

Central to this discussion has been a very serious attempt to present the basis of quantum measurement in terms that I hope are understandable regardless of our comfort with the mathematics. Here I have again clarified the difficulties that exist in our claims to know. Where we cannot perform measurements we cannot claim knowledge in a scientific sense. Speculation based on judgment is always appropriate. I believe, however, that any speculation must be undertaken with humility and awareness of the limitations of science.

For example, in my discussion of measurement I have studiously avoided any direct use of the wave function, collapse of the wave function, or wave-particle duality. These are unnecessary and often lead to confusion because of our human tendency to imagine reality.

I have ended with some discussion of the problems associated with a formulation of human consciousness. There are great difficulties in this perhaps greatest of all problems. Although I do not expect anything resembling an easy resolution in satisfying detail, I am of the opinion that physics is open enough to provide at least a substantial part of that resolution. Any clarification here holds the promise of deep rewards, because an understanding of the brain and consciousness may bring us closer to a comprehension of God's interaction with us. We must, however, remain aware of Tillich's contention that God is infinite mystery.

There are distinct differences between what I have said here and the position expressed by Lothar Schäfer in his contribution to this discussion (Schäfer 2006a, b, c, d). I understand that he is presenting a vision. That vision in part is an intense acknowledgment of the transcendence of God. I also claim a belief in that transcendence—but I am concerned that we move carefully, acknowledging the present state of our scientific knowledge and the basis for that. We may be prepared to accept metaphysical positions that are contrary to the positions of theoretical physicists, but we have the responsibility to be aware of the points at which we differ from theoretical physics and to justify the basis of that difference.

Schäfer seems to accept a duality or separate existence for mind and what loosely may be termed the physical or material. I am more in agreement with the position Ervin Laszlo (2006) takes that the physical and mental are aspects of one and the same reality. However, even though this general position may be correct, I am not yet prepared to claim it as a solution to the problem of human consciousness. History may prove Laszlo correct in his claim that "An electron acts the way it does because in addition to its physical pole it also has a mental pole, and humans act the way they do because in addition to a brain they have mind and consciousness" (2006, 540). To me this seems too quick a resolution to the questions of our understanding of both matter and of consciousness. We may, in the future, encounter a complementarity principle for the physical and the mental. But the complementarity principle did not tell us that the electron had a particle and a wave pole. The resolution was not an ontological clarification of the electron. The clarification was epistemological.

Schäfer seems to accept the wave function as a reality. His objective apparently is to offer a picture of matter that is different from the common concept of matter. In my discussion of quantum measurement I have tried to provide the basis for our understanding of what may be termed matter. There I have studiously avoided considering the wave function in any part of the actual measurement. The wave function, as Born pointed out, has no physical reality. To emphasize this aspect of the quantum theory I have pointed out that our calculation of quantum numbers is carried out based on symmetries of the Hamiltonian operator; no actual representation of the state vector as a wave function is ever needed. Our understanding of elementary particles is based on the symmetries of mathematical groups. I do not apologize for the fact that the picture of physical reality presented by the physicist is less than that hoped for. This apparent limitation may hold a deeper understanding for us as we investigate human consciousness.

Schäfer presents plots of constant values of the probability density $|\Psi|^2$ for the electron in the hydrogen atom. These are very helpful as long as we are aware that they are representations not of the electron but of the probability of finding the electron in regions of space. These probability densities are no more physically real than the wave function, from which they are obtained. Even the actual measurement of an electron in orbit is beyond physical possibility.

Schäfer proposes a conception of virtual quantum states as transcendent reality. Although I believe that transcendence is an important aspect of any serious discussion of religion and science, I fail to see unoccupied quantum states as evidence of transcendence. Mathematically these result from whatever interactions we use to write the Hamiltonian. This is not to claim that we can describe those interactions in common language, particularly when they involve entities such as the spin or hyperspin, with no classical analogs. We also cannot decide by any measurement which state is occupied and which is not. All we can measure is the result of a transition. We measure spectra, which must be understood in terms of statistical mechanics. In this the quantum states and their properties, for example symmetries of the Hamiltonian, are simply necessary parts of the mathematical picture on which we base our understanding of laboratory measurements. In this sense the states are real ingredients of our mathematics, and statistical mechanics only provides probabilities of their occupation.

In his discussions of the DNA molecule Schäfer brings forth what seems to be a foundational part of his vision. He claims first that molecular states may be thought to exist in the virtual cosmic state space before the corresponding molecules exist as actual lumps of matter. "Chances are," he writes, "that the quantum states that actualize in DNA already existed at a time when real DNA molecules did not yet exist as material lumps on this planet" (2006a, 512). I understand this to mean that life on Earth already existed in the mind of God before it became a reality on the surface of the planet. That is a stirring vision provided we are ready to claim an understanding of what is meant by the mind of God. Scientifically we have no way to investigate the consequences of this claim. Specifically physics attributes no meaning to the existence of molecular quantum states before interactions are present. We must acknowledge this in our discussion.

There are deep mysteries in modern physics. These begin, perhaps, with such things as the photon and the spin. The photon is not a localizable particle but a quantum of the electromagnetic field. The spin is the source of magnetism through an interaction we call spin exchange. But for this our physical picture is only derivable from the mathematical operators emerging from perturbation theory. And we discuss virtual photons and quasiparticles in our considerations of electromagnetic interactions. Even our picture of heat in a solid, which involves the concept of a phonon, the quantum of vibration, is not transparent. Each step we take reveals more of the great depth of mystery in the universe, even as it provides certain answers. And we can say nothing about the form in which those answers may come before an investigation is made. I have tried to bring some of this out in the discussion here, but I can only touch small parts. I acknowledge that in this great dialogue in which we are engaged we find what George F. R. Ellis (2003) has called intimations of transcendence. But this also is not without mystery. I am cautioning us in our enthusiasm.

At the end of his response to Laszlo, Schäfer cites a manifesto published by Le Monde, which was signed by a group of highly regarded scientists that includes Schäfer. Although I would prefer to read the entire manifesto before committing myself in any absolute way, I can concur with most of the content of the sentences Schäfer presents. We often contend, as scientists, that we should not allow our religious or metaphysical ways of thinking to, a priori, influence our ordinary practice of science. They do, of course, as we see in Einstein's resistance to the quantum theory. I also agree that we must, a posteriori, reflect on the philosophical, ethical, and metaphysical implications of science. These dramatically exceed the topic of our present discussion when one includes the adjective *ethical*. As a physicist I am acutely aware of the necessity of ethical considerations. In some instances our reflections should be a priori rather than a posteriori. The present discussion, in comparison, appears benign.

I also agree that to fail to undertake these considerations isolates us from society. This isolation benefits neither the society we serve nor our science.

I believe, as well, that the dialogue between religion and science is one of the most important that can occupy our efforts and our talents. Many of us affirm that in science we are encountering an intimation of transcendence. When we encounter transcendence we do ask questions that take us beyond the laboratory. In asking those questions we must, however, remain clear about their consistency or lack of consistency with contemporary scientific thought.

In his response to Laszlo, Schäfer claims that what he is presenting is not science. Although I have pointed to the inconsistencies of some of Schäfer's ideas with present theoretical physics, I do not want to brush his vision from the table. In the quantum theory we are encountering something that potentially is far more revealing of the depths of the universe than we presently recognize. I believe that the investigations of consciousness will bring us into contact with something more. And so I am grateful to Schäfer for expressing his ideas and opening them for discussion.

NOTES

1. The squares of the moduli of α and β are equal to the probabilities provided the state vector is normalized, that is, constructed so that its magnitude is unity.

2. For example, the resting potential of neurons, which is the electrical potential of the interior of the neuron with respect to the exterior, is of the order of -100 millivolts, and the insulating thickness of the cell membrane is about 2.3 nanometers. The electric field within the membrane is then of the order of 50 million volts per meter.

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