Review

Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World. By Amir Alexander. New York: Scientific American/Farrar, Straus and Giroux, 2014. 352 pp. Softcover $27.00.

Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World is a marvelous book about a particular moment in mathematics history when thinking about geometric objects as the "sum" of their infinitesimally thin cross-sections revolutionized the way we thought about those objects and eventually led to the discovery of calculus. Amir Alexander tells this story through a clever pitting of some very strange bedfellows, the Jesuit order and Thomas Hobbes, against the early adopters of infinitesimals. For those of us weary of the science versus faith wars, we might perhaps be forgiven for assuming the worst concerning Alexander's use of the word "dangerous" in the title of his book. It hints of yet another heroic tale of open-minded scientists fighting closed-minded clerics in a battle for the scientific future. Some readers will undoubtedly draw precisely this conclusion from the book, but this turns out *not* to be Alexander's project. His real motivation for describing the seventeenth-century battles over infinitesimals—aside from the sheer delight in sharing their development—appears to be to convince us of the true danger: the tyranny of axiomatic reasoning. Alexander wants to argue that the kind of inductive reasoning so usefully employed by our mathematical forebears in the development of infinitesimals is friendlier to scientifically progressive democratic institutions than the rigorous deductive reasoning one finds in axiomatic approaches to mathematics.

Infinitesimals are notoriously difficult mathematical entities to describe. At the founding of calculus they were famously mocked by George Berkeley as the "ghosts of departed quantities." So difficult are they to describe with any precision that they were not put on a rigorous foundation until the 1960s, receiving a full treatment in Abraham Robinson's 1966 book *Non-Standard Analysis*. Infinitesimals are essentially quantities of arbitrarily small but positive size which can safely be treated as having *no* size once a certain amount of algebraic manipulation has been completed. Mathematicians used them frequently and intuitively—if not rigorously—to great profit in the early development of calculus.

The Jesuit part of the story constitutes Part I of Alexander's book and it begins unfortunately and unpromisingly in the tone of a Dan Brown novel: "On August 10, 1632, five men in flowing black robes came together in a somber Roman palazzo on the left bank of the Tiber River. . . . Their mission: to pass judgment upon the latest scientific and philosophical ideas of the age."

An orthodox Catholic reader braces himself for the worst, perhaps wondering only whether or not Tom Hanks will be available when the filming begins. But any resemblance to Dan Brown both begins and ends in that paragraph. Alexander's description of the Jesuit order is largely generous, the occasional lapse all the more

 ISSN 0591-2385

striking precisely because those lapses are so rare (e.g., they are "highly educated and *fanatical*").

That being said, quite a few Jesuits play the role of bad guy in this story and my only problem with the way Alexander handles them is the problem of quantity. Sometimes Alexander uses the phrase "the Jesuits" to mean merely the Revisors General (a five-member advisory body charged with the examination of Jesuit teaching and publications); sometimes "the Jesuits" are a handful of well-placed Jesuits whose positions command respect; by my count only once does "the Jesuits" mean the general congregation (and of course even this representative body does not include all Jesuits). We are warned about the enormous power Jesuit leaders wield, given that their order prizes obedience and given that their leaders have a proven ability to destroy careers. But for all this worrisome potential "the Jesuits" are surprisingly impotent. It is a credit to Alexander's honest storytelling that this impotence is glaring even as a handful of Jesuits manage to silence prominent Italian supporters of infinitesimals. Remember those five Jesuits we met on the left bank of the Tiber River in 1632? They did indeed forbid certain teachings about infinitesimals *within the order* that year and their successors did so again—as Alexander documents—in 1641, 1643, and 1649—presumably in the same menacing black robes. Infinitesimals keep coming up *within the Jesuit order*, precisely because they appear to be very useful mathematically and because—like all good scholars—Jesuit researchers were constantly refining the notion of infinitesimal, hoping to find one that could be confidently taught as true. The fact that infinitesimals wouldn't be put on a rigorous foundation until the 1960s makes impressive the tenacity with which Jesuit researchers pursued them; it also makes the reluctance of the Jesuit leadership to proclaim anything incontrovertibly true about infinitesimals appear wise in retrospect.

In Part II of *Infinitesimal*, we shift north to England where we are introduced to Thomas Hobbes and his battles with one of the great early adopters of infinitesimals, John Wallis. In addition to their delicious squabbling, there's an important reason why Hobbes's opposition to the use of infinitesimals rates his inclusion in Alexander's book. It's Hobbes's insistence that firm axiomatic foundation must precede the use of any mathematical concept. Alexander wants to convince us that this insistence is potentially dangerous, not merely because insisting on axiomatic rigor might retard scientific progress, but because axiomatic reasoning is a form of reasoning which tries to *compel* its participants to certain conclusions. The theorems of an axiom system might well be called "propositions," but have no doubt about it, it is naïve to think that the propositions of an axiom system are merely being *proposed*. When the reader of a proof gets to its end and assents to the QED, there is no more choice in the matter: the acceptance of truth is now *compelled*.

It is in this context that Alexander examines Hobbes's political writing, in particular *Leviathan*. Hobbes thoroughly admired the axiomatic structure of Euclid's *Elements* and built his political theory in *Leviathan* axiomatically: carefully defining terms, turning basic observations about "the state of nature" into axioms, and then reasoning his way to theorems about the political system best suited to human survival. Not human flourishing (whatever that might mean), but survival. Hobbes made no appeal to authority as in scholastic tradition, but rather looked to the data of nature as his source of authority. *Leviathan*'s thoroughgoing axiomatic materialism was a pioneering way of doing political philosophy. The conclusion that he reached, however—that an absolute monarch best suited human needs for security and order—was appalling to many of Hobbes's contemporaries (and certainly to us). More alarming for Alexander is that Hobbes should want to *compel* his readers to these conclusions through the power of axiomatic reasoning.

Alexander's use of Hobbes as a character in the story of infinitesimals is thus an inspired choice. We might be tempted at the end of Part I to simply conclude that religion—and religion alone—is the enemy of scientific progress. We learn from Alexander, however, that Hobbes—no great friend of religion—too rode sallies against the remarkably useful infinitesimals, and for the same reason that the Jesuit Revisors General did: infinitesimals lacked a firm axiomatic foundation. It is not religion *per se* that posed the threat to infinitesimals back in the day—and scientific progress now—but those who would insist on the primacy of axiomatic reasoning. This is a very provocative claim.

Axiomatic reasoning was at one time thought to be the very model for scientific reasoning. It is now widely recognized that inductive reasoning is more proper to the sciences, and rightly so. But does that mean that in this very scientific of ages, three hundred years since infinitesimals' first inductive skirmishes with axiomatic reasoning, that inductive reasoning in mathematics has now thoroughly gained the upper hand, relegating the axiomatic approach to a mere historical curiosity? Well no. The axiomatic method in mathematics has never been stronger, richer or more interesting. Have infinitesimals and the kind of inductive reasoning associated with them been "dangerous" to axiomatic reasoning? In the sense that a messenger can bring very unwelcome tidings, the early effectiveness of infinitesimals was certainly unsettling to axiomatic thinking. But if the battle for infinitesimals represented a more science-friendly inductive revolt against the tyranny of axiomatic reasoning, it is a rebellion that has been put down. Infinitesimals themselves were put on an axiomatic footing in the 1960s. Game, set, and match to axiomatic reasoning.

There are other quibbles one might make with Alexander along religious lines, including his inadequate treatment of the theological differences between Catholics and Protestants. In an effort to emphasize the more democratic character of Protestants over Catholics, he mischaracterizes, for example, the differences between their theologies of priesthood and grace. In that same vein he tries to argue in his closing pages that the hierarchical bias of Catholic thought held Italy back from continuing its leadership in science, while tacitly conceding that this same Catholic Italy had a leadership in science that could be lost. All societies appear to gain and lose momentum in various cultural endeavors. It has been argued, for example, that the England Alexander so much admires lost a century of dominance in mathematics to a foolish attachment to Newton's more awkward version of the calculus, instead of adopting the Leibniz version that had swept the continent and whose superior notation survives to this day. But these are all quibbles.

Amir Alexander has written a wonderful book, one that I enjoyed even more on a second reading. Describing challenging mathematics for a lay audience is no easy task, but Alexander does this remarkably well. I found myself eagerly pulling out a pencil and paper to follow Alexander's descriptions on a number of occasions. Even when Alexander is wrong—and I think his main thesis is quite wrong—I find him wrong in very interesting ways. I heartily recommend this book.

JOHN JOSEPH SCHOMMER
John Schommer is a Professor of Mathematics in the Department of Mathematics and Statistics, University of Tennessee at Martin, Martin, TN 38238, USA
jschomme@utm.edu